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Educational program for determining the inertial characteristics of surfaces

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1. ABSTRACT

In the current context of the intensive development of online education due to the health crisis, it is necessary to develop educational packages to support students. This article develops an algorithm based on numerical methods to calculate the moments of inertia of flat surfaces with complex shapes. The article presents the theoretical aspects as well as the main steps of the algorithm for calculating the moments of inertia for flat surfaces. The Matlab program described in the paper can be used on e-learning platforms to train students. The computational algorithm is implemented in the Matlab programming environment which is familiar in the academic environment. The program can be used to discipline Strength of Materials.

2.THEORETICAL ASPECTS

By definition the moments of inertia of surfaces are:

-The moment of inertia about the axis x and y relative to the xOy system:

$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$

-Centrifugal moment of inertia relative to the xOy system:

$$I_{xy} = \int_A xy dA$$

-The main directions:

$$\alpha_1 = \frac{1}{2} arctg\left(-\frac{2I_{xy}}{I_x - I_y}\right); \alpha_2 = \alpha_1 + \frac{\pi}{2}$$

-Principal moments of inertia:

$$I_{1,2} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I^2{}_{xy}}$$

3.IMPLEMENTATION OF NUMERICAL ALGORITHM IN THE MATLAB PROGRAMMING ENVIRONMENT

1. Discretization of the surface into triangles (figure 2);
2. Calculation area, center of gravity and the inertial characteristics for each triangle of discretization (figure 1):

$$A_i = \frac{1}{2}\begin{vmatrix} x_{1i} & y_{1i} & 1 \\ x_{2i} & y_{2i} & 1 \\ x_{3i} & y_{3i} & 1 \end{vmatrix}; i = 1 \ldots n. \qquad x_{Ci} = \frac{x_{1i} + x_{2i} + x_{3i}}{3}; \; y_{Ci} = \frac{y_{1i} + y_{2i} + y_{3i}}{3}$$

$$I_{xi} = \frac{A}{6}\left(y_{1i}(y_{1i} + y_{2i}) + y_{2i}(y_{2i} + y_{3i}) + y_{3i}(y_{3i} + y_{1i})\right)$$

$$I_{yi} = \frac{A}{6}\left(x_{1i}(x_{1i} + x_{2i}) + x_{2i}(x_{2i} + x_{3i}) + y_{3i}(x_{3i} + x_{1i})\right)$$

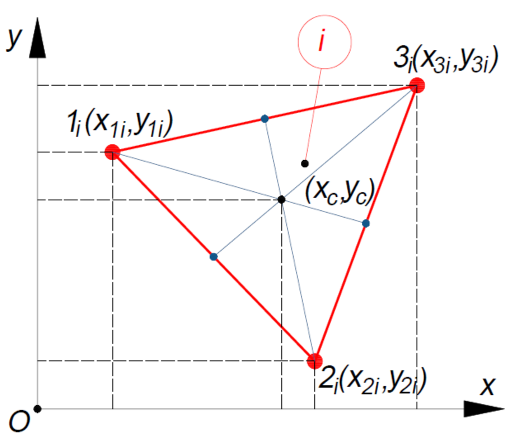

Figure 1

3. Calculation of axial moments of inertia and centrifugal moment of inertia by summation on all triangles by Steiner rule:

$$I_x = \sum_{i=1}^{n} I_{xi}; I_y = \sum_{i=1}^{n} I_{yi}; I_{xy} = \sum_{i=1}^{n} I_{xiyi};$$

$$I_x = I_{xC} + Ay_C^2; I_{xC} = I_x - Ay_C^2;$$

$$I_y = I_{yC} + Ax_C^2; I_{xC} = I_x - Ax_C^2;$$

$$I_{xy} = I_{xyC} + Ax_C^2 y_C^2; I_{xyC} = I_{xy} - Ax_C^2 y_C^2$$

where,

I_x, I_y, I_{xy} moments of inertia in relation to xOy

I_{xC}, I_{yC}, I_{xyC} moments of inertia about the axes passing through the center of gravity

4. Calculation of main directions and main moments of inertia and the equivalent rectangle (figure 3).

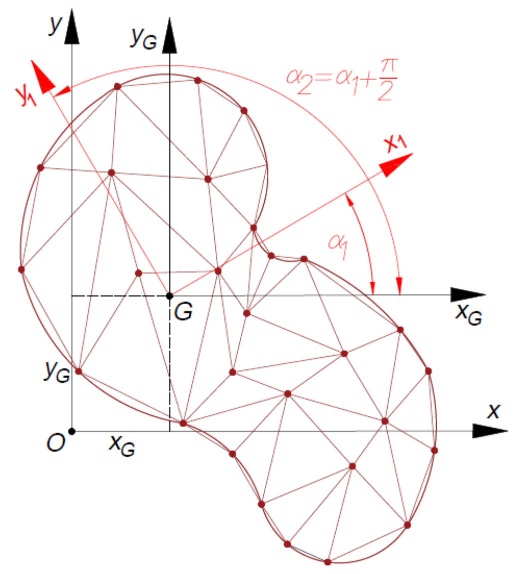

Figure 2. Meshing the surface.

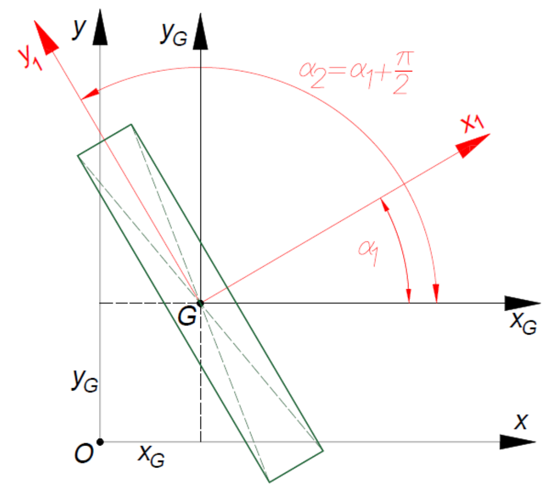

Figure 3. Meshing the surface.

4. CONCLUSIONS

In the figure 4 and 5 show two analyzes performed with the proposed algorithm:

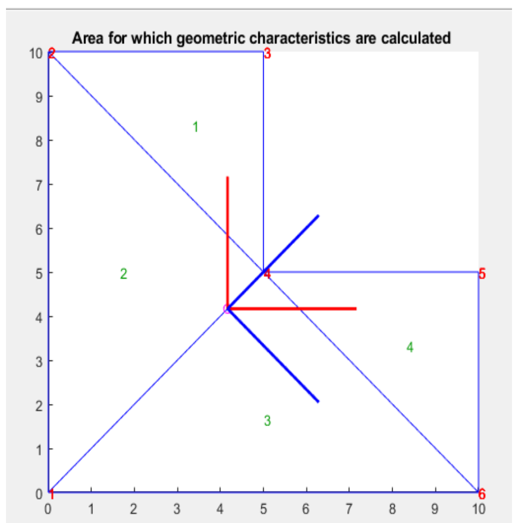

Figure 4. Moments of inertia for the shape surface L

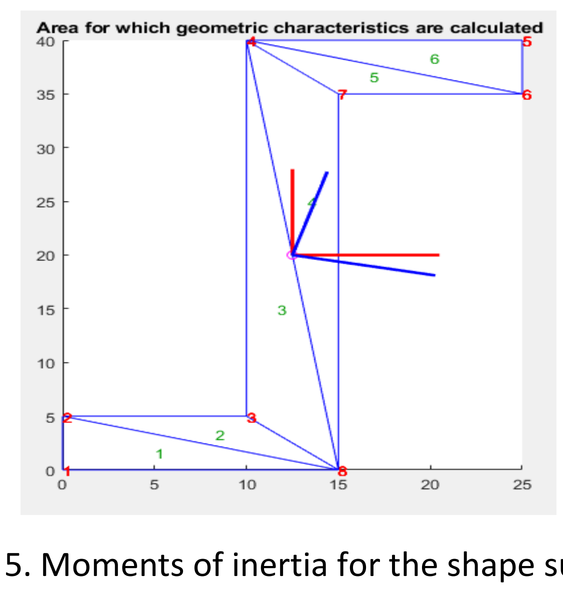

Figure 5. Moments of inertia for the shape surface Z

- The results obtained with the program Matlab were validated by analytical calculation for the analyzed sections;
- The calculation program allows the determination of the inertial characteristics for complex surfaces;
- The program allows the optimization of the shape and dimensions of the parametrically defined cross sections;
- It is possible to calculate surfaces that have curved or straight-line boundaries;
- The program allows the equivalence of a complex section with a rectangular section oriented according to the main directions and having extreme axial moments of inertia after these directions.

References:

[1] Mechanics of Materials 6th Edition, Russell C. Hibbeler. Publisher- Prentice Hall, 2004.
[2] https://www.mathworks.com/products/matlab.html